kd education academy (9582701166)

Time : 3 Hour | STD 11 Science Maths
kd90+ ch-1 sets | Total Marks : 100

*** Choose the right answer from the given options. [1 Marks Each]** **[50]**

1. Choose the correct answers from the given four option:
In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Then, the number of students who play neither is.

(A) 0 (B) 25 (C) 35 (D) 45

Ans. :

d. 25

Solution:

Total number of students = 60

Number of students who play cricket = 25

Number of students who play tennis = 20

Number of students who play cricket and tennis both = 10

$\Rightarrow n(C \cap T) = 10$

$\therefore n(C \cap T) = n(C) + n(T) - n(C \cap T)$

$= 25 + 20 - 10 = 45 - 10 = 35$

$\therefore n(C' \cap T') = n(U) - n(C \cap T)$

$= 60 - 35 = 25$

Hence, the correct option is (b).

2. Let n(A) = 28,n(A ∩ B) =8, n(A∪B) =52, then n(A ∩ B′):

(A) 30 (B) 32 (C) 20 (D) none of these

Ans. :

c. 20

Solution:

Given n(A) = 28, n(A ∩ B) = 8.

We have A ∩ B′ = A - A ∩ B.

This give n(A ∩ B′) = n(A) -n(A ∩ B)

or, n(A ∩ B′) = 28 - 8 = 20.

3. The solution set of 3x − 4 < 8 over the set of non-negative square numbers is:

(A) {1, 2, 3} (B) {1,4} (C) {1} (D) {16}

Ans. :

c. {1}

Solution:

3x − 4 < 8

3x < 12

$x < 4$

Hence set of non-negative square numbers belonging to the above set is {1}.

4. In a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone?

(A) 35 (B) 48 (C) 60 (D) 22.

Ans. :

c. 60.

Solution:

Let M, P and C denote the sets of students who have opted for mathematics, physics, and chemistry, respectively.

Here,

$n(M) = 100,\ n(P) = 70,\ n(C) = 40$

Now,

$n(M \cap P) = 30, n(M \cap C) = 28,$

$n(P \cap C) = 23, n(M \cap P \cap C) = 18$

Number of students who opted for only mathematics:

$n(M \cap P' \cap C)' = \{M \cap (P \cap C)'\}$

$= n(M) - n\{M \cap (P \cap C)\}$

$= n(M) - n\{(M \cap P) \cup (M \cap C)\}$

$= n(M) - \{n(M \cap P) + n(M \cap C) - n(M \cap P \cap C)\}$

$= 100 - (30 + 28 - 18)$

$= 60$

$\therefore$ the number of students who opted for mathematics alone is 60.

5. The set of all those elements of A and B which are common to both is called:

(A) Union of two sets (B) Intersection of two sets

(C) Disjoint sets (D) None of these

Ans. :

b. Intersection of two sets

Solution:

The set of all those elements of A and B which are common to both is called A intersection B = A ∩ B.

6. Choose the correct answers from the given four option:

If X and Y are two sets and X′ denotes the complement of X, then $X \cap (X \cup Y)'$ is equal to.

(A) X. (B) Y. (C) ϕ. (D) $X \cap Y$.

Ans. :

b. ϕ.

Solution:

Let $x \in X \cap (X \cup Y)'$

$\Rightarrow x \in X \cap (X' \cup Y)'$

$\Rightarrow x \in (X \cap X) \cap (X \cap Y')$

$\Rightarrow x \in \phi \cap (x \cap Y') \left[\because A \cap A' = \phi\right]$

$\Rightarrow x \in \phi$

Hence, the correct option is (c).

7. The number of subsets of a set containing n elements is:

(A)	(B)	(C)	(D)
n	$2^n - 1$	n^2	2^n.

Ans. :

d. 2^n.

Solution:

The total number of subsets of a finite set consisting of n elements is 2^n.

8. If A and B are two disjoint sets, then $n(A \cup B)$ is equal to:

(A) $n(A) + n(B)$ (B) $n(A) + n(B) - n(A \cap B)$

(C) $n(A) + n(B) + n(A \cap B)$ (D) $n(A)\, n(B)$.

Ans. :

a. $n(A) + n(B)$.

Solution:

Two sets are disjoint if they do not have a common element in them, i.e., $A \cap B = \phi$.

$\therefore n(A \cup B) = n(A) + n(B)$.

9. In a city 20% of the population travels by car 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus is:

(A) 80% (B) 40% (C) 60% (D) 70%.

Ans. :

c. 60%.

Solution:

Suppose C and B represents the population travels by car and bus respectively.

$n(C \cup B) = n(C) + n(B) - n(B \cap C)$

$= 0.20 + 0.50 - 0.10$

$= 0.6 \text{ or } 60\%$.

10. IF R = {(2, 1), (4, 3), (4, 5)}, then range of the function is?

(A) Range R = {2, 4}

(B) Range R = {1, 3, 5}

(C) Range R = {2, 3, 4, 5}

(D) Range R {1, 1, 4, 5}

Ans. :

b. Range R = {1, 3, 5}

Solution:

Given R = {(2, 1), (4, 3), (4, 5)}

then Range (R) = {1, 3, 5}

11. If A = {2, 4, 6, 8, 10}, B = {1, 3, 5, 7, 9}, then A - B =_____:

(A) {}

(B) {2, 4, 6, 8, 10}

(C) {1, 3, 5, 7, 9}

(D) None

Ans. :

b. {2, 4, 6, 8, 10}

Solution:

A = {2, 4, 6, 8, 10}

B = {1, 3, 5, 7, 9}

A - B = {2, 4, 6, 8, 10} - {1, 3, 5, 7, 9} = {2, 4, 6, 8, 10}

12. A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 420 consumers like product B.Then, the least number of consumers that must have liked both the products is:

(A) 140

(B) 180

(C) 210

(D) 190

Ans. :

a. 140

Solution:

Total consumers = 1000

Like product A = n(A) = 720

Like product B = n(B) 420

n (A ∩ B) (Both the products) = n(A) + n(B) − n(A ∪ B)

= 720 + 420 − 1000

= 140

13. Two finite sets have m and n elements. The number of elements in the power set of first set is 48 more than the total number of elements in power set of the second set. Then, the values of m and n are:

(A) 7, 6

(B) 6, 3

(C) 7, 4

(D) 3, 7.

Ans. :

c. 6, 4.

Solution:

ATQ:

$2^m - 1 = 48 + 2^n - 1$

$\Rightarrow 2^m - 2^n = 48$

$\Rightarrow 2^m - 2^n = 2^6 - 2^4$

By comparing we get:

m = 6 and n = 4.

14. In an examination 80% passed in English, 85% in Maths, 75% in both and 40 students failed in both subjects. Then the number of students appeared are:

(A) 300 (B) 400 (C) 500 (D) 600

Ans. :

b. 400

Solution:

$n(E) = 80$

$n(M) = 85$

$n(E \cap M) = 75$

$n(E \cup M) = n(E) + n(M) - n(E \cap m)$

$= 80 + 85 - 75 = 90$

$n(E \cup M)' = 10$

Let n be the total number of students appeared.

$\frac{10}{100} \times n = 40$

$\therefore n = 400$

15. In a certain group of 36 people, 18 are wearing hats and 24 are wearing sweaters. If six people are wearing neither a hat nor a sweater, then how many people are wearing both a hat and a sweater?

(A) 30 (B) 22 (C) 12 (D) 8

Ans. :

c. 12

Solution:

Since 6 people are wearing neither hat nor sweater

$n(H \cup S) = 36 - 6 = 30$

By set theory

$n(H \cap S) = n(H) + n(S) - n(H \cup S)$

$= 18 + 24 - 30 = 12$

16. Choose the correct answers from the given four option:

A survey shows that 63% of the people watch a News Channel whereas 76% watch another channel. If x% of the people watch both channel, then

(A) x = 35 (B) x = 63 (C) $39 \leq x \leq 63$ (D) x = 39

Ans. :

c. $39 \leq x \leq 63$

Solution:

Let p% of the people watch a channel and q% of the people watch another channel

$\because n(p \cap q) = x\%$ and $n(p \cup q) \leq 100$

So, $n(p \cap q) \geq n(p) + n(q) - n(p \cap q)$

$100 \geq 63 + 76 - x$

$100 \geq 139 - x \Rightarrow x \geq 139 - 100 \Rightarrow x \geq 39$

Now n(p) = 63

$\therefore n(p \cap q) \leq n(p) \Rightarrow x \geq 63$

So $39 \leq x \geq 63$. Hence, the correcr option is (c).

17. There are 19 hockey players in a club. On a particular day 14 were wearing the prescribed hockey shirts, while 11 were wearing the prescribed hockey pants. None of then was without hockey pant or hockey shirt. How many of them were in complete hockey uniform?

(A) 8 (B) 6 (C) 9 (D) 7

Ans. :

b. 6

Solution:

We can look at it in 2 ways

First by set theory

n(A ∩ B) = n(A) + n(B) − n(A ∪ B)

= 14 + 11 − 19 = 6

Qualitatively, we know that 14 people are wearing prescribed hockey shirts,which leaves us with 5 players who must be wearing hockey pants. So out of 11 players who are wearing hockey pants, 5 are not wearing hockey shirts while the other 6 are in complete uniform.

18. The cardinality of the set P(P(P(f))) is.

(A) 0 (B) 1 (C) 2 (D) 4

Ans. :

d. 4

19. The set $(A \cup B')' \cup B \cap C$ is equal to:

(A) $A' \cup B \cup C$ (B) $A' \cup B$ (C) $A' \cup C'$ (D) $A' \cap B$.

Ans. :

b. $A' \cup B$.

Solution:

$(A \cup B')' \cup (B \cap C)$

$= [A \cap (B')'] \cup (B \cap C)$ (De Morgen law)

$= (A' \cap B) \cup (B \cap C)$

$= (A' \cup C) \cup B$ (Distributive law)

Disclimer: The question seems to be incorrect or there is some printing mistake in the question. The options given in the question does not match with the answer.

20. In a class of 50 students, 10 did not opt for math, 15 did not opt for science and 2 did not opt for either. How many students of the class opted for both math and science.

(A) 24 (B) 25 (C) 26 (D) 27

Ans. :

d. 27

Solution:

Total students = 50

Students who did not opt for math = 10

Students who did not opt for Science = 15

Students who did not opt for either maths or science = 2

Total of 40 students in math and 13 did not opt for science but did for math = 40 - 13 = 27

So, students of the class opted for both math and science is 27

21. If A = (6, 7, 8, 9), B = (4, 6, 8, 10) and C = {x : x ∈ N : 2 < x ≤ 7} ; find :B − C

(A) {4, 6} (B) {4, 6, 8} (C) {6, 8, 10} (D) {8, 10}

Ans. :

d. {8, 10}

Solution:

C = {3, 4, 5, 6, 7}

B − C = {4, 8, 10}

22. If A and B are two sets such that $n(A) = 70$, $n(B) = 60$, $n(A \cup B) = 110$, then $n(A \cap B)$ is equal to:

(A) 240 (B) 50 (C) 40 (D) 20.

Ans. :

d. 20.

Solution:

We have:

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 70 + 60 - 110$

$= 20.$

23. In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, whose numbers are divisible by 5 opt for Chemistry and those

whose numbers are divisible by 7 opt for Math. How many opt for none of the three subjects?

(A) 19 (B) 41 (C) 21 (D) 57

Ans. :

b. 41

Solution:

The number of students who took at least one of the three subjects can be found by finding out $A \cup B \cup C$,

where A is the set of those who took Physics, B the set of those who took Chemistry and C the set of those who opted for Math.

Now

$A \cup B \cup C = A + B + C - (A \cap B + B \cap C + C \cap A)(A \cap B \cap C)$

A is the set of those who opted for Physics $= \frac{120}{2} = 60$ Students

B is the set of those who opted for Chemistry $= \frac{120}{5} = 24$

C is the set of those who opted for Math $= \frac{120}{7} = 17$

The 10th, 20th, 30th..... numbered students would have opted for both Physics and Chemistry.

Therefore $A \cap B = \frac{120}{10} = 12$

The 14th, 28th, 42nd..... Numbered students would have opted for Physics and Math.

Therefore, $C \cap A = \frac{120}{14} = 8$

The 35th, 70th.... numbered students would have opted for Chemistry and Math.

Therefore, $B \cap C = \frac{120}{35} = 3$

And the 70th numbered student would have opted for all three subjects.

Therefore, $A \cup B \cup C = 60 + 24 + 17 - (12 + 8 + 3) + 1 = 79$

Number of students who opted for none of the three subjects = 120 – 79 = 41

24. Choose the correct answers from the given four option:

Let S = {x | x is a positive multiple of 3 less than 100} P = {x | x is a prime number less than 20}. Then n(S) + n(P) is.

(A) 34 (B) 31 (C) 33 (D) 30

Ans. :

b. 31

Solution:

Given that: S = {x | x is a positive multiple of 3 < 100}

∴ S = {3, 6, 9, 12, 15 18, ..., 99}

n(S) = 33

T = (x | x is a prime number < 20)

$\therefore$ T = {2, 3, 5, 7, 11, 13, 17, 19}

n(T) = 8

So, n(S) + n(T) = 33 + 8 = 41

Hence, the correct option is (b).

25. Out of 500 first year students, 260 passed in the first semester and 210 passed in the second semester.If 170 did not pass in either semester, how many passed in both semesters?

(A) 30 (B) 40 (C) 70 (D) 140

Ans. :

d. 140

Solution:

Let A be the set of students who passed first semester so n(A) = 260

and B be the set of students who passed second semester so n(B) = 210.

Now 170 did not passed any semester.

So, (500 − 170 = 330) students passed atleast one of the semesters.

$\therefore$ n(A∪B) = 330

Now n(A ∪ B) = n(A) + n(B) − n(A ∩ B)

330 = 260 + 210 − n(A ∩ B)

n(A ∩ B) = 140

26. If A, B and C are any three sets, then $A - (B \cup C)$ is equal to.

(A) $(A - B) \cup (A - C)$ (B) $(A - B) \cup C$

(C) $(A - B) \cap C$ (D) $(A - B) \cap (A - C)$

Ans. :

d. $(A - B) \cap (A - C)$

Solution:

Given A, B and C are any three sets.

Now $A - (B \cup C) = (A - B) \cap (A - C)$

27. In a group of 15,7 have studied German, 8 have studied French, and 3 have not studied either. How many of these have studied both German and French?

(A) 0 (B) 3 (C) 4 (D) 5

Ans. :

b. 3

Solution:

Since 3 have neither studied German nor French

n(G ∪ F) = 15 − 3 = 12

By set theory

n(G ∩ F) = n(G) + n(F) − n(G ∪ F)

= 7 + 8 − 12 = 3

28. Two finite sets have N and M elements. The number of elements in the power set of first set is 48 more than the total number of elements in power set of the second test. Then the value of M and N are.

(A) 7, 6 (B) 6, 4 (C) 7, 4 (D) 6, 3

Ans. :

b. 6, 4

Solution:

Let A and B be two sets having m and n numbers of elements respectively

Number of subsets of A = 2m

Number of subsets of B = 2n

Now, according to question

2m - 2n = 48

⇒ 2n(2m - n - 1) = 24(22 - 1)

So, n = 4 and m - n = 2

⇒ m – 4 = 2

⇒ m = 2 + 4

⇒ m = 6

29. In a science talent examination, 50% of the candidates fail in Mathematics and 50% fail in Physics. If 20% fail in both these subjects, then the percentage who pass in both Mathematics and Physics is:

(A) 0% (B) 20% (C) 25% (D) 50%

Ans. :

b. 20%

Solution:

By set theory

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$ where M and P are sets of students failing in respective subjects.

= 0.5 + 0.5 − 0.2 = 0.8

This indicates 80% of the class fails in at least one of the given subjects while 20% pass in both.

30. Choose the correct answers from the given four option:

If X = {8n - 7n - 1 | n ∈ N} and Y = {49n - 49 | n ∈ N}. Then

(A) $X \subset Y$ (B) $Y \subset X$ (C) $X = Y$ (D) $X \cap Y = \phi$

Ans. :

a. $X \subset Y$

Solution:

X = {8n - 7n - 1| n ∈ N} = {0, 49, 490, ...}

Y = {49n - 49 | n ∈ N} = {0, 49, 147, ..., 490, ...}

Clearlut, every element of X is in Y but every element of Y is not in X.

$\therefore X \subset Y$

31. If A = {1, 2, 3, 4, 5}, then the number of proper subsets of A is:

(A) 31 (B) 38 (C) 48 (D) 54

Ans. :

a. 31

32. All the students of a batch opted Psychology, Business, or both. 73% of the students opted Psychology and 62% opted Business. If there are 220 students, how many of them opted for both Psychology and business?

(A) 60 (B) 100 (C) 77 (D) 35

Ans. :

c. 77

Solution:

By set theory

n(P ∩ B) = n(P) + n(B) − n(P ∪ B)

= 0.73 + 0.62 − 1.00 = 0.35

35% of 220 = 77

33. Find the equivalent set for A − B.

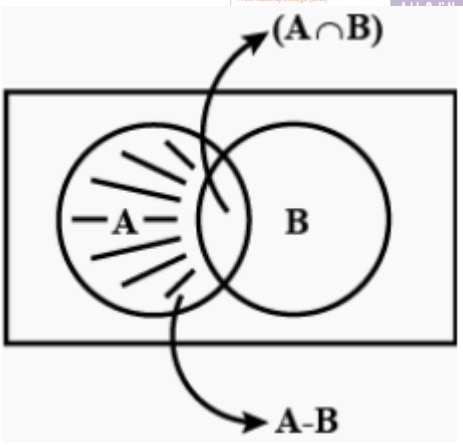
(A) $A \cup (A \cap B)$ (B) $A - B$ (C) $A - (A \cap B)$ (D) $A \cap B$

Ans. :

c. $A - (A \cap B)$

Solution:

Hence By this graph we see that $A - B = A - (A \cap B)$



34. In a community of 175 persons, 40 read the Times, 50 read the Samachar and 100 do not read any. How many persons read both the papers?

(A) 10 (B) 15 (C) 20 (D) 25

Ans. :

b. 15

Solution:

Since 100 do not read any
$n(T \cup S) = 175 - 100 = 75$
y set theory
$n(T \cap S) = n(T) + n(S) - n(T \cup S)$
$= 40 + 50 - 75 = 15$

35. In an examination, 34% of the candidates fail in Arithmetic and 42% in English.If 20% fail in Arithmetic and English, the percentage of those passing in both subjects is:

(A) 44 (B) 45 (C) 46 (D) 47

Ans. :

a. 44

Solution:

$n(A) = 34$
$n(B) = 42$
$n(A \cap B) = 20$
$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 34 + 42 - 20 = 56$
$n(A \cup B)' = 100 - n(A \cup B) = 100 - 56 = 44$

36. In set-builder method the null set is represented by:

(A) $\{\}$ (B) ϕ (C) $\{x : x \neq x\}$ (D) $\{x : x = x\}$.

Ans. :

c. $\{x : x \neq x\}$.

37. If out of 150 students who read at least one newspaper The Times of India, The Hindustan Times and The Hindu. There are 65 who read The Times of India, 41 who read The Hindu and 50 who read The Hindustan Times.What is the maximum possible number of students who read all the three newspaper?

(A) 7 (B) 42 (C) 3 (D) Cannot be determined

Ans. :

c. 3

Solution:

$a + b + c = 150$
$a + 2b + 3c = 156$
Hence $b + 2c = 6$
To maximise c we take minimum value of b that is 0.
Hence $c = 3$

38. If n(A) denotes the number of elements in set A and if n(A) = 4,n(B) = 5 and $n(A \cap B) = 3$ then $n[(A \times B) \cap (B \times A)]$=

(A) 8 (B) 9 (C) 10 (D) 11

Ans. :

b. 9

Solution:

For (A × B) ∩ (B × A) we have to do the mapping of A × B or B × A between common elements.

no. of ways of mapping will be 3 × 3 = 9

n[(A × B) ∩ (B × A)] = 9

39. Given A = {a, b, c, d, e, f, g, h} and B = {a, e, i, o, u} then B - A is equal to:

(A) {i, o, u} (B) {a, b, c} (C) {c, d, e} (D) {a, i, z}

Ans. :

a. {i, o, u}

Solution:

The sets A = {a, b, c, d, e, f, g, h} and B = {a, e, i, o, u}, in order to find the difference between the two sets as B–A, we begin by writing all the elements of B and then take away every element of A which is also the element of B. Since B share the elements a, e with A, so B - A = {i, o, u}.

40. A and B are two sets having 3 and 5 elements respectively and having 2 elements in common. Then the number of elements in A × B is:

(A) 6 (B) 36 (C) 15 (D) None of these

Ans. :

c. 15

Solution:

Total ordered pairs =n(A) × n(B) = 3 × 5 = 15.

41. If A = {x : x is a multiple of 3} and B = {x : x is a multiple of 5}, then A - B is:

(A) $A \cap B$ (B) $A \cap \overline{B}$ (C) $\overline{A} \cap \overline{B}$ (D) $\overline{A \cap B}$.

Ans. :

b. $A \cap \overline{B}$.

Solution:

A = {x : x is a multiple of 3}

A = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, ...

B = {x : x is a multiple of 5}

B = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...

Now, we have:

A - B = 3, 6, 9, 12, 18, 21, 24, 27, 30, 33,36, 39, 42, ...

$= A \cap \overline{B}$.

42. If A = {1, 2, 3, 4, 5}, then the number of proper subsets of A is:

(A) 120 (B) 30 (C) 31 (D) 32.

Ans. :

c. 31.

Solution:

The number of proper subsets of any set is given by the formula 2n - 1, where n is the number of elements in the set.

Here,

n = 5

∴ Number of proper subsets of A = 25 - 1 = 31.

43. An investigator interviewed 100 students to determine the performance of three drinks: milk, coffee and tea. The investigator reported that 10 students take all three drinks milk, coffee and tea; 20 students take milk and coffee; 25 students take milk and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of three drinks is:

(A) 10 (B) 20 (C) 25 (D) 30.

Ans. :

b. 20.

Solution:

solve for None:

80 + None = 100

None = 20.

44. Let U be the universal set containing 700 elements. If A, B are subsets of U such that $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then, $n(A' \cap B') =$

(A) 400 (B) 600 (C) 300 (D) None of these.

Ans. :

c. 300.

Solution:

$n(A' \cap B') = n(A \cup B')$

$= n(U) - n(A \cup B)$

$= 700 - 200 + 300 - 100 = 300.$

45. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Then the number of subsets of A containing exactly two elements is:

(A) 20 (B) 40 (C) 45 (D) 90

Ans. :

c. 45

Solution:

Number of elements in A = 10

Number of subsets of A containing exactly two elements

= Number of ways we can select 2 elements from 10 elements

$10c_2 = \frac{10 \times 9}{2} = 45$

∴ Number of subsets of A containing exactly two elements = 45

46. If A and B are two sets such that n(A) = 17, n(B) = 23, n(A ∪ B) = 38, find n(A ∩ B):

(A) 1 (B) 2 (C) 3 (D) 4

Ans. :

b. 2

Solution:

We know,

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$n(A \cap B) = 17 + 23 - 38 = 2$

47. Let A and B be two sets such that n(A) = 16, n(B) = 12, and n(A ∩ B) = 8.Then n(A∪B) equals:

(A) 28 (B) 20 (C) 36 (D) 12

Ans. :

b. 20

Solution:

$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 16 + 12 - 8 = 20$

48. The set $(A \cup B')' \cup B \cap C$ is equal to:

a. $A' \cup B \cup C$

b. $A' \cup B$

c. $A' \cup C'$

d. $A' \cap B$.

Ans. :

b. $A' \cup B$.

Solution:

$(A \cup B')' \cup (B \cap C)$

$= [A \cap (B')'] \cup (B \cap C)$ (De Morgen law)

$= (A' \cap B) \cup (B \cap C)$

$= (A' \cup C) \cup B$ (Distributive law)

Disclimer: The question seems to be incorrect or there is some printing mistake in the question. The options given in the question does not match with the answer.

49. Let A and B be two sets that $n(A) = 16$, $n(B) = 14$, $n(A \cup B) = 25$. Then, $n(A \cap B)$ is equal to:

a. 30

b. 50

c. 5

d. None of these.

Ans. :

c. 5.

Solution:

We know:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Now,

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 16 + 14 - 25$

$= 5.$

50. If A = {1, 3, 5, 7, 9, 11, 13, 15, 17} B = {2, 4, ..., 18} and N the set of natural numbers is the universal set, then $A' \cup (A \cup B) \cup B')$ is

a. ϕ

b. N

c. A

d. B

Ans. :

b. N

Solution:

Given that:

A = {1, 3, 5, 7, 9, 11, 13, 15, 17}

B = {2, 4, ..., 18}

U = N = {1, 2, 3, 4, 5, ...}

$A' \cup (A \cup B) \cap B' = A'[(A \cap B') \cup (B \cap B')]$

$= A' \cup (A \cap B') \cup \phi \quad [\because A \cap A' = \phi]$

$= A' \cup (A \cap B')$

$= (A' \cup A) \cap (A' \cup B')$

$= N \cup (A' \cup B') \quad [\because A' \cup A = N]$

$= A' \cup B'$

$= (A \cup B') = (\phi)' = N \quad [\because A \cap B = \phi]$

Hence, the correct option is (b).

*** Answer the following questions in one sentence. [1 Marks Each]** **[9]**

51. Describe the following sets in Roster form:

$\{x \in N : x^2 < 25\};$

Ans. : In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces { }. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

$1 \in N \because 1^2 = 1 < 25$

$2 \in N \because 2^2 = 4 < 25$

$3 \in N \because 3^2 = 9 < 25$

$4 \in N \because 4^2 = 16 < 25$

Hence, the above set can be written as {1, 2, 3, 4}.

52. If $X = \{8^n - 7x - 1 : n \in N\}$ and $Y = \{49(n-1) : n \in N\}$, then prove thet $X \subseteq Y$.

Ans. : $X = \{8^n - 7x - 1 : n \in N\}$

$Y = \{49(n-1) : n \in N\}$

In order to show that $X \subseteq Y$ we shoe the every element of X is an element of Y.

So let $x \in X \Rightarrow x = 8^n - 7m - 1$ for same $m \in N$

$\Rightarrow x = (1+7)^m - 7m - 1$

$= ({}^mC_0 1^m + {}^m C_1 1^{m-1} 7 + \ldots + {}^mC_{m-1} 1^1 7^{m-1} + {}^m C_m 7^m) - 7m - 1$ [using binomail expansion]

$= 1 + 7m + {}^m C_2 7^2 + {}^m C_3 7^3 + \ldots + {}^mC_m 7^m - 7m - 1$

$= {}^mC_2 7^2 + {}^mC_3 7^3 + \ldots + {}^mC_m 7^m$

$= 49({}^mC_2 + {}^m C_3 + \ldots + {}^mC_m 7^m),\ m \geq 2$

$= 49t_m,\ m \geq 2,$ where $t_m = {}^m C_2 + {}^m C_3 7 + \ldots + {}^mC_m 7^{m-2}$

Is some positive integer depending on $m \geq 2$

For $m = 1$

$x = 1^8 - 7 \times 1 - 1$

$= 8 - 8$

$= 0$

Hence, X contains all positive integral multiples of 49.

Also, Y consistes all positive integral multiples of 49, including 0, for n = 1.

Thuse, we coclude that $X \subseteq Y$.

53. The given following sets are finite & in which of it infinite in if?

$\{x \in N : x > 5\}$

Ans. : Finite, $\because \{x \in N : x > 5\}$ = {6, 7, 8,...} which is infinite.

54. If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8}, C = {7, 8, 9, 10, 11}, and D = {10, 11, 12, 13, 14}. Find:

$A \cup C$

Ans. : $A \cup C = \{x : x \in A\ x \in C\}$

= {1, 2, 3, 4, 5, 7, 8, 9, 10, 11}.

55. The given set is the example of an empty set or not?

Set of all even prime numbers.

Ans. : As 2 belongs to this set, so it is non-empty.

56. If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8}, C = {7, 8, 9, 10, 11}, and D = {10, 11, 12, 13, 14}. Find:

$A \cup B$

Ans. : A = {1, 2, 3, 4, 5}

B = {4, 5, 6, 7, 8}

So, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

= {1, 2, 3, 4, 5, 6, 7, 8}.

57. List all the elements of the following sets:

$B = \left\{x : x = \frac{1}{2n-1}, 1 \leq n \leq 5\right\}$;

Ans. : Let's find the values of $x = \frac{1}{2n-1}, \text{for } 1 \leq n \leq 5$

for n = 1, $x = \frac{1}{1} = 1$

for n = 2, $x = \frac{1}{2\times2-1} = \frac{1}{4-1} = \frac{1}{3}$

for n = 3, $x = \frac{1}{2\times3-1} = \frac{1}{6-1} = \frac{1}{5}$

for n = 4, $x = \frac{1}{2\times4-1} = \frac{1}{8-1} = \frac{1}{7}$

for n = 5, $x = \frac{1}{2\times5-1} = \frac{1}{10-1} = \frac{1}{9}$

Hence, $B = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right\}$.

58. Are the following sets equal?

A = {x : x is a letter in the word paper}, B = {x : x is a letter in the word paper},

C = {x : x is a letter in the word paper}.

Ans. : A = {a, e, p, r}

B = {a, e, p, r} (repetition of 'p' is not allowed)

C = {a, o, , r}

as A = B $\neq$ C, ∴ The sets are not equal.

59. The given following sets are finite & in which of it infinite in it?

Set of concentric circles in a plane.

Ans. : Infinite, since with a common centre infinitely many circles can be drown in a plane.

*** Given section consists of questions of 2 marks each.** **[12]**

60. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee.

Ans. : Let T be the set of students who like tea and C be the set of students who like coffee.

Here n(T) = 150, m (C) = 225 and $n(C \cap T) = 100$

We know that $n(C \cup T) = n(C) + n(T) - n(C \cap T)$

= 150 + 225 - 100 = 275

∴Number of students taking either tea or coffee += 275

∴Number of students taking neither tea nor coffee = 600 - 275 = 325

61. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers.
Find the number of people who read at least one of the newspaper.

Ans. : Here

n(U) = a + b + c + d + e + f + g + h = 60 ....(i)

n (H) = a + b + c +d = 25 ....(ii)

n(T) = b + c + f + g = 26 .....(iii)

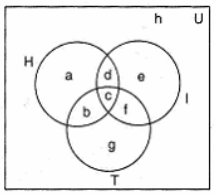
n(I) = c + d + e + f = 26 ....(iv)

$n(H \cap I)$ = c + d = 9 .....(v)

$n(H \cap T)$ = b + c = 11 .....(vi)

$n(T \cap I)$ = c + f = 8 ....(vii)

$n(H \cap T \cap I)$ = c = 3 ....(viii)



Putting value of c in (vii),

3 + f = 8 $\Rightarrow$ f = 5

Putting value of c in (vi),

3 + b = 11 $\Rightarrow$ b = 8

Putting values of c in (v),

3 + d = 9 $\Rightarrow$ d = 6

Putting value of c, d, f in (iv),

3 + 6 + e + 5 = 26 $\Rightarrow$ e = 26 - 14 = 12

Putting value of b, c, f in (iii),

8 + 3 + 5 + g = 26 $\Rightarrow$ g = 26 - 16 = 10

Putting value of b, c, d in (ii)

a + 8 + 3 + 6 = 25 $\Rightarrow$ a = 25 - 17 = 8

Number of people who read at least one of the three newspapers

= a + b + c + d + e + f + g

= 8 + 8 + 3 + 6 + 12 + 5 + 10 = 52

62. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers.
Find the number of people who read exactly one newspaper.

Ans. : Here

n(U) = a + b + c + d + e + f + g + h = 60 ....(i)

n(H) = a + b + c +d = 25 ....(ii)

n(T) = b + c + f + g = 26 .....(iii)

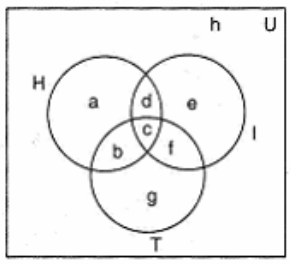
n(I) = c + d + e + f = 26 ....(iv)

$n(H \cap I)$ = c + d = 9 ....(v)

$n(H \cap T)$= b + c = 11 .....(vi)

$n(T \cap I)$ = c + f = 8 .....(vii)

$n(H \cap T \cap I)$ = c = 3 ....(viii)



Putting value of c in (vii),

3 + f = 8 $\Rightarrow$ f = 5

Putting value of c in (vi),

3 + b = 11 $\Rightarrow$ b = 8

Putting values of c in (v),

3 + d = 9 $\Rightarrow$ d = 6

Putting value of c, d, f in (iv),

3 + 6 + e + 5 = 26 $\Rightarrow$ e = 26 - 14= 12

Putting value of b, c, f in (iii),

8 + 3 + 5 + g = 26 $\Rightarrow$ g = 26 - 16= 10

Putting value of b, c, d in (ii)

a + 8 + 3 + 6 = 25 $\Rightarrow$ a = 25 - 17 = 8

Number of people who read exactly one newspapers

= a + e + g

= 8 + 12 + 10 = 30

63. Given L = {1, 2, 3, 4}, M = {3, 4, 5, 6} and N = {1, 3, 5}
Verify that $\mathrm{L} - (\mathrm{M} \cup \mathrm{N}) = (\mathrm{L} - \mathrm{M}) \cap (\mathrm{L} - \mathrm{N})$

Ans. : Given L, = {1, 2, 3, 4}, M = {3, 4, 5, 6} and N = {1, 3, 5}

$\mathrm{M} \cap \mathrm{N} = \{1, 3, 4, 5, 6\}$

$\mathrm{L} - (\mathrm{M} \cap \mathrm{N}) = \{2\}$

Now, L-M = {1, 2} and L-N = {2, 4}

$\{\mathrm{L} - \mathrm{M}) \cap \{\mathrm{L} - \mathrm{N}) = \{2\}$

Hence, $\mathrm{L} - \{\mathrm{M} \cup \mathrm{N}) = \{\mathrm{L} - \mathrm{M}) \cap (\mathrm{L} - \mathrm{N})$.

64. Write the following sets in the roaster form:

$\mathrm{E} = \left\{\mathrm{w} | \frac{\mathrm{w}-2}{\mathrm{w}+2} = 3, \mathrm{w} \in \mathrm{R}\right\}$

Ans. : $\mathrm{E} = \left\{\mathrm{w} | \frac{\mathrm{w}-2}{\mathrm{w}+2} = 3, \mathrm{w} \in \mathrm{R}\right\}$

$\therefore \frac{\mathrm{w}-2}{\mathrm{w}+2} = 3$

$\Rightarrow 3\mathrm{w} + 9 = \mathrm{w} - 2$

$\Rightarrow 3w - w = -2 - 9$

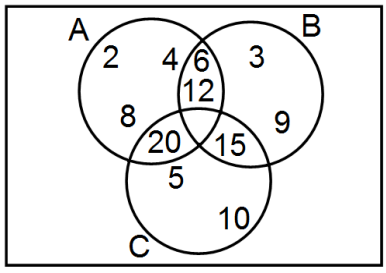
$\Rightarrow 2w = -11$

$\Rightarrow w = \frac{-11}{2} \in R$

Hence, $E = \left\{\frac{-11}{2}\right\}$

65. A, B and C are subsets of Universal Set U. If A = {2, 4, 6, 8, 12, 20} B = {3, 6, 9, 12, 15}, C = {5, 10, 15, 20} and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.

Ans. : Given that: A, B, and C are the subsets of a universal set U.



Where A = {2, 4, 6, 8, 12, 20} B = {3, 6, 9, 12, 15} and C = {5, 10, 15, 20}.

*** Given section consists of questions of 3 marks each.** **[9]**

66. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only?

Ans. : Here

n (A) = a + b + c + d = 21 ....(i)

n(B) = b + c + f + g = 26 ....(ii)

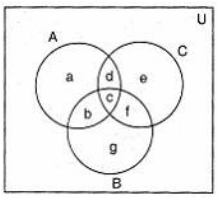
n(C) = c + d + e + f = 29 ....(iii)

$n(A \cap B)$ = b + c = 14 ....(iv)

$n(C \cap A)$ = c + d = 12 ....(v)

$n(B \cap C)$ = c + f = 14 ....(vi)

$n(A \cap B \cap C)$ = c = 8 ....(vii)



Putting value of c in (iv), (v) and (vi)

b + 8 = 14 $\Rightarrow$ b = 6

8 + d = 12 $\Rightarrow$ d = 4

8 + f = 14 $\Rightarrow$ f = 6

Putting value of c, d, f in (iii),

8 + 4 + e + 6 = 29 $\Rightarrow$ e = 29 - 18 = 11

Number of people who like product C only = 11

67. A college awarded 38 medals in Football, 15 in Basketball and 20 in Cricket. If these medals went to a total of 58 men and only three men got medals in all three sports, then how many received medals in exactly two of the three sports.

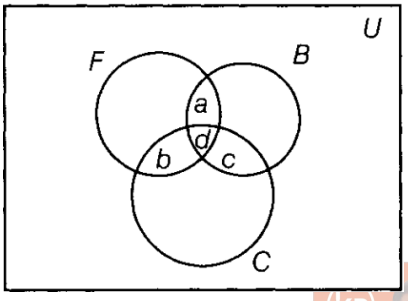
Ans. : Suppose $n(F), n(B)\ and\ n(C)$ denote the number of men who received medals in Football, Basketball and Cricket, respectively. Then,

n(F) = 38, n(B) = 15, n(C) = 20, n($F \cup B \cup C$) = 58 and n($F \cap B \cap C$) = 3

$\because n(F \cup B \cup C)$= n(F) + n(B) + n(C) + $n(F \cap B \cap C) - n(F \cap B) - n(F \cap C) - n(B \cap C)$

58 = 38 + 15 + 20 + 3$-n(F \cap B) - n(F \cap C) - n(B \cap C)$

$\Rightarrow n(F \cap B) + n(F \cap C) + n(B \cap C)$ = 76 - 58 = 18



Here, 'a' = the number of men who got medals in Football and Basketball only.

'b' = the number of men who got medals in Football and Cricket only.

'c' = the number of men who got medals in Basketball and Cricket only.

'd '= the number of men who got medals in all the three games.

Thus, d = $n(F \cap B \cap C)$ = 3

and $n(F \cap B) + n(F \cap C) + n(B \cap C)$ = 18

$\Rightarrow$ (a + d) + (b + d) + (c + d) = 18

$\Rightarrow$ a + b + c + 3d = 18

$\Rightarrow$ a + b + c+ 3(3) = 18 [Put d = 3, given]

$\therefore$ a + b + c = 9

Hence, people who got medals in exactly two of the three sports is 9.

68. Let $T = \left\{x \middle| \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$. Is T an empty set? Justify your answer.

Ans. : Given that: $T = \left\{x \middle| \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$

$\Rightarrow \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}$

$\Rightarrow \frac{(x+5)-5(x-7)}{x-7} = \frac{4x-40}{13-x}$

$\Rightarrow \frac{x5-5x+35}{(x-7)} = \frac{4x-40}{13-x}$

$\Rightarrow \frac{-4x+40}{(x-7)} = \frac{4x-40}{13-x}$

$\Rightarrow -4(x-10)(12-x) = 4(x-10)(x-7)$

$\Rightarrow -4(x-10)(13-x+x-7) = 0$

$\Rightarrow -4(x-10)6 = 0$

$\Rightarrow -24(x-10)=0$

$\Rightarrow x-10=0$

$\Rightarrow x=10$

$\therefore T=10$

Hence, T is not an empty set.

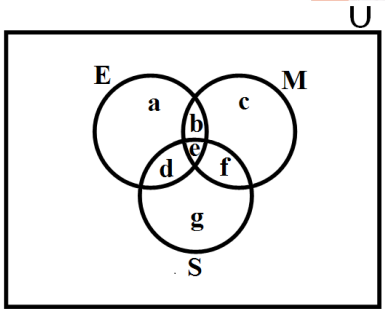
*** Given section consists of questions of 5 marks each.** **[20]**

69. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science; 4 in English and Science; 4 in all the three. Find how many passed.
    i. In English and Mathematics but not in Science.
    ii. In Mathematics and Science but not in English.
    iii. In Mathematics only.
    iv. In more than one subject only.

Ans. : Let the ser of student who passed in Mathematics be M, the set of student who passed in English be E and the set of studnet who passed in Science be S.

Then m(U) = 100, n(M) = 12, n(E) = 15, n(S) = 8, $n(E\cap M)=6, n(M\cap S)=7, n(E\cap S)=4$ and $n(E\cap M\cap S)=4$

Let us draw a Venn diagram



According to the Venn diagram

$n(E\cap M\cap S)=4\Rightarrow e=4$

$n(E\cap M)=6\Rightarrow b+e=6$

$\Rightarrow b+4=6\Rightarrow b=2$

$n(M\cap S)=7\Rightarrow e+f=7$

$\Rightarrow 4+f=7\Rightarrow f=3$

$n(E\cap S)=4\Rightarrow d+e=4$

$\Rightarrow d+4=4\Rightarrow d=0$

n(E) = 15 ⇒ a + b + d + e = 15 ⇒ a + 2 + 0 + 4 = 15 ⇒ a = 9

n(M) = 12 ⇒ b + c + e + f = 12 ⇒ 2 + c + 4 +3 = 12 ⇒ c = 3

n(S) = 8 ⇒ d + e + f + g = 8 ⇒ 0 + 4 + 3 + g = 8 ⇒ g = 1

Hence we get,

i. Number of students who passed in English and Mathematics but not in Science, b = 2.

ii. Number of students who passed in Mathematics and Science but not in English, f = 3.

iii. Number of students who passed in Mathematics only, c = 3.

iv. Number of studnets who passed in mote than one subject = b + e + d + f = 2 + 4 + 0 + 3 = 9.

70. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find

i. The number of families which buy newspaper A only.

ii. The number of families which buy none of A, B and C.

Ans. : Let A be the set of families which buy newspaper A, B be the set of families which buy newspaper B and C be the set of families which buy newspaper C. The Then, n(U) = 1000, n(A) = 40%, n(B) = 20% and n(C) = 10%, $n(A \cap B) = 5\%, n(B \cap C) = 3\%, n(A \cap C) = 4\%$ and $n(A \cap B \cap C) = 2\%$

i. Percentage of families which buy newspaper A only

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= (40 - 5 - 4 + 2)\% = 30\%$

$\therefore$ Number of families which buy newspaper A only $= 1000 \times \frac{33}{100} = 3300$

ii. Percentage of families which buy none of A, B and C

$= n(U) - n(A \cup B \cup C)$

$= n(U) - [n(A) + n(B) + n(C) - n(A \cup B) - n(B \cap C)$

$- n(A \cap C) + n(A \cap B \cap C)]$

$= 100 - [40 + 20 + 10 - 5 - 3 - 4 + 2]$

$= 100 - 60 = 40\%$

Number of families which buy none of A, B and C $= 10000 \times \left(\frac{40}{100}\right) = 4000$

71. Match the following sets for all sets A, B and C.

(i)	$((A' \cup B') - A)'$	(a)	$A - B$
(ii)	$[B' \cup (B' - A)]'$	(b)	A
(iii)	$(A - B) - (B - C)$	(c)	B
(iv)	$(A - B) \cap (C - B)$	(d)	$(A \times B) \cap (A \times C)$
(v)	$A \times (B \cap C)$	(e)	$(A \times B) \cup (A \times C)$
(vi)	$A \times (B \cup C)$	(f)	$(A \cap C) - B$

Ans. :

(i)	$((A' \cup B') - A)'$	(b)	A
(ii)	$[B' \cup (B' - A)]'$	(c)	B
(iii)	$(A - B) - (B - C)$	(a)	$A - B$

(iv)	$(A-B)\cap(C-B)$	(f)	$(A\cap C)-B$
(v)	$A\times(B\cap C)$	(d)	$(A\times B)\cap(A\times C)$
(vi)	$A\times(B\cup C)$	(e)	$(A\times B)\cup(A\times C)$

Solution:

1. $((A'\cup B')-A)'=[(A'\cup B')\cap A)']\ [\because A-B=A\cap B']$

$=[(A\cap B)'\cap A)']'\ [\because A'\cup B'=(A\cap B)']$ $=[(A\cap B)']'\cup(A')'\ [\because (A')'=A]$

$=(A\cap B)\cup A=A$. So (i) is mathc with (b).

2. $[B'\cup(B'-A)]'=[B'\cup(B'\cap A')]'\ [\because A-B=A\cap B']$

$=(B')'\cap(B'\cap A')'\ [\because A'\cap B'=(A\cup B)']$ $=B\cap(B\cup A)=B$ So (ii) is matched with (c).

3. $(A-B)-(B-C)=(A\cap B')-(B\cap C')\ [\because A-B=(A\cap B')]$

$=(A\cap B')\cap(B\cap C')'$ $=(A\cap B')\cap(B\cap C')')\ [(A\cap B)'=A'\cup B']$ $=(A\cap B')\cap(B'\cap C)$

$=[A\cap(B'\cup C)]\cap[B'\cap(B'\cup C)]$ $=[A\cap(B'\cup C)]\cap B'$ $=(A\cap B')\cap(B'\cup C)\cap B'$

$=(A\cap B')\cap B'=(A\cap B')=A-B$ So, (iii) is matched with (a).

4. $(A-B)\cap(C-B)=(A\cap B')\cap(C\cap B')\ [\because A-B=(A-B')]$

$=(A\cap C)\cap B'=(A\cap C)-B\ [\because A\cap B'=A-B]$ Hence, (iv) is matched with (f).

5. $A\times(B\cup C)=(A\times B)\cap(A\times C)$

So, (v) is mathced with (d).

6. $A\times(B\cup C)=(A\times B)\cup(A\times C)$

So, (vi) is matched with (e).

72. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows:

French = 17, English = 13, Sanskrit = 15

French and English = 09, English and Sanskrit = 4

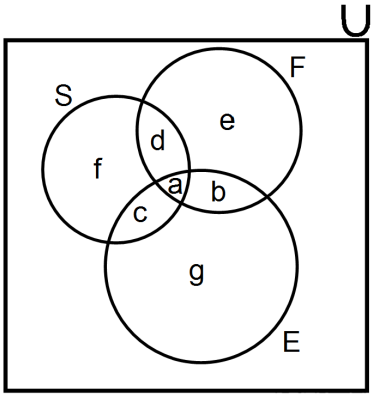
French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study.

i. French only.
ii. English only.
iii. Sanskrit only.
iv. English and Sanskrit.
v. French and Sanskrit but not English.
vi. French and English but not Sanskrit.
vii. At least one of the three languages but not French.
viii. None of the three languages.

Ans. : Let F be the set of students who study French, E be the set of students who study English and S be the set of students who study Sanskrit.

Then, n{U) = 50, n(F) =17, n{E) = 13, and n{S) = 15,

$n(F \cap E) = 9, n(E \cap S) = 4, n(F \cap S) = 5, n(F \cap E \cap S) = 3$



From the figure, we have

$a = n(E \cap F \cap S) = 3$

$a + d = n(F \cap S) = 5 \ \therefore d = 2$

$a + b = n(F \cap E) = 9 \ \therefore b = 6$

$a + c = n(S \cap E) = 4 \ \therefore c = 1$

a + b + d + e = n(F) = 17 or 3 + 6 + 2 + e = 17 $\therefore$ e = 6

a + b + c + g = n(E) = 13 or 3 + 6 + 1 + g = 13 $\therefore$ g = 3

a + c + d + f = n(S) = 15 or 3 + 1 + 2 + f = 15 $\therefore$ f = 9

i. Number of students studying French only = e = 6

ii. Number of students studying English only = g = 3

iii. Number of students studying Sanskrit only = f = 9

iv. Number of students studying English and Sanskrit but not French = c = 1

v. Number of students studying French and Sanskrit but not English = d = 2

vi. Number of students studying French and English but not Sanskrit = b = 6

vii. Number of students studying at least one of the three languages = a + b + c + d + e + f + g = 30

viii. Number of students studying none of the three languages but not French = 50 - 30 = 20.

----- the journey of thousands miles begins with a single step -----